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NLL QED CORRECTIONS TO DEEP INELASTIC SCATTERING *

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The $O(\alpha^2 \log(Q^2/m_e^2))$ leptonic QED corrections to unpolarized deeply inelastic electron-nucleon scattering are calculated in the mixed variables.

1 Introduction

Deep inelastic scattering provides us with detailed information on the nucleon structure. In order to extract the parton distribution functions from DIS cross sections and to measure $\alpha_s(M_Z^2)$ with high precision it is crucial to control the QED radiative corrections. The 1– and 2–loop leading–log QED corrections were derived in Ref. [1–3]. Complete 1-loop corrections for DIS were given in Refs. [4]. Furthermore the universal leading logarithmic corrections were derived to $O((\alpha L)^5)$ both for polarized and unpolarized processes in [5], where also the resummation of the $O((\alpha \ln^2(z))^k)$ for polarized scattering was given. In this paper, we summarize our recent results of NLO leptonic QED corrections in mixed variables [7].

2 Mixed variables

In general radiative corrections do strongly depend on how the kinematic variables are measured. In this paper, we consider the case of mixed variables, i.e. $y = y_h$ is measured from the hadron side and $Q^2 = Q_l^2$ is measured from the lepton side. Then the rescaled variables for initial and final state radiation are given by

$$\begin{aligned} \mathsf{ISR} \ : \quad & \widehat{y} = \frac{y_h}{z}, \ \widehat{Q}^2 = zQ_l^2, \ \widehat{S} = zS, \ \widehat{x} = zx_m, \\ & J^I(z) = 1, \qquad z_0^I = \max\left\{y_h, Q_0^2/Q_l^2\right\} \ , \end{aligned} \tag{1}$$

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$$\text{FSR} \; : \quad \widehat{y} = y_h, \; \widehat{Q}^2 = \frac{Q_l^2}{z}, \; \widehat{S} = S, \; \widehat{x} = \frac{x_m}{z} \; ,$$
$$J^F(z) = \frac{1}{z} \; , \qquad z_0^F = x_m \; . \tag{2}$$

Here $J^{I,(F)}(z)$ are the Jacobians for initial (final) state radiation and z_0 denotes the lower bound of the rescaling variable. Q_0^2 is introduced as a cut on Q_h^2 to keep the process duly deep inelastic, i.e. to avoid significant contributions of the Compton peak. In the subsequent section, we frequently use the following shorthand notation for a function with rescaled variables in its argument :

$$\widetilde{F}_{I,F}(y, Q^2) = F\left(y = \widehat{y}_{I,F}, Q^2 = \widehat{Q}^2_{I,F}\right), \tag{3}$$

where I, F indicate ISR and FSR rescaling.

3 NLO corrections

We parameterize the k-th order differential cross section as

$$\frac{d^2\sigma^{(k)}}{dy_h dQ_l^2} = \sum_{l=0}^{k} \left(\frac{\alpha}{2\pi}\right)^k \ln^{k-l}\left(\frac{Q^2}{m_e^2}\right) C^{(k,l)}(y, Q^2) \; , \tag{4}$$

with $C^{(0,0)}(y, Q^2)$ denoting Born cross section. $C^{(1,0)}(y, Q^2)$ and $C^{(2,0)}(y, Q^2)$ were calculated in [3]. The $O(\alpha)$ non-logarithmic term $C^{(1,1)}(y, Q^2)$ was derived in Ref. [4]. We re-calculated these corrections [7] and agree with the previous results.

NLO corrections $C^{(1,1)}(y, Q^2)$ are obtained using RG equations for mass factorization and charge renormalization. This method was first implemented in [6] for initial state corrections to e^+e^- annihilation, a single differential cross section in the s–channel. We deal with double–differential distributions for a t–channel process. At first the scattering cross section is decomposed as follows :

$$\frac{d^2\sigma}{dy_h dQ_l^2} = \frac{d^2\sigma^0}{dy_h dQ_l^2} \otimes \left\{ \Gamma^I_{ee} \otimes \hat{\sigma}_{ee} \otimes \Gamma^F_{ee} + \Gamma^I_{\gamma e} \otimes \hat{\sigma}_{e\gamma} \otimes \Gamma^F_{ee} + \Gamma^I_{ee} \otimes \hat{\sigma}_{\gamma e} \otimes \Gamma^F_{e\gamma} \right\} \tag{5}$$

with $\Gamma_{ij}^{I,F}(z,\mu^2/m_e^2)$ the initial and final state operator matrix elements and $\hat{\sigma}_{kl}(z,Q^2/\mu^2)$ the respective Wilson coefficients which obey the representations

$$\begin{aligned}
\Gamma_{ij}^{I,F}\left(z,\frac{\mu^2}{m_e^2}\right) &= \delta(1-z)+\sum_{m=1}^{\infty}\left(\frac{\alpha}{2\pi}\right)^m\sum_{n=0}^{m}\Gamma_{ij}^{I,F(m,n)}(z)\ln^{m-n}\left(\frac{\mu^2}{m_e^2}\right) &(6)\\
\hat{\sigma}_{kl}\left(z,\frac{Q^2}{\mu^2}\right) &= \delta(1-z)+\sum_{m=1}^{\infty}\left(\frac{\alpha}{2\pi}\right)^m\sum_{n=0}^{m}\widehat{\sigma}_{kl}^{(m,n)}(z)\ln^{m-n}\left(\frac{Q^2}{\mu^2}\right)\ , &(7)
\end{aligned}$$

where $j(l)$ denotes the incoming and $i(k)$ the outgoing particle. In the cross section (5), the μ^2–dependences cancel each other and the final expression expands in $\alpha/2\pi$ and $\ln(Q^2/m^2)$, to $O(\alpha^2 L)$. There are several contributions to NLO corrections :

i LO initial and final state radiation off $C_{ee}^{(1,1)}(y,Q^2)$

ii coupling constant renormalization of $C_{ee}^{(1,1)}(y,Q^2)$

iii LO initial state splitting of $P_{\gamma e}$ at $C_{e\gamma}^{(1,1)}(y,Q^2)$

iv LO final state splitting of $P_{e\gamma}$ at $C_{\gamma e}^{(1,1)}(y,Q^2)$

v NLO initial and final state radiation off $C_{ee}^{(0,0)}(y,Q^2)$.

The first contribution $C_{\mathsf{i}}^{(2,1)}(y,Q^2)$ is

$$\begin{aligned}
C_{\mathsf{i}}^{(2,1)}(y,Q^2) &= \int_0^1 dz P_{ee}^0\left[\theta(z-z_0^I)J^I\widetilde{C}_I^{(1,1)}(y,Q^2)-C^{(1,1)}(y,Q^2)\right] &(8)\\
&+\int_0^1 dz P_{ee}^0\left[\theta(z-z_0^F)J^F\widetilde{C}_F^{(1,1)}(y,Q^2)-C^{(1,1)}(y,Q^2)\right],
\end{aligned}$$

where $P_{ee}^0(z)$ is the LO splitting function:

$$P_{ee}^0(z)=\frac{1+z^2}{1-z}\ . \tag{9}$$

The QED coupling is renormalized as

$$\alpha(\mu^2)=\alpha(m_e^2)\left[1-\frac{\beta_0}{4\pi}\ln\left(\frac{\mu^2}{m_e^2}\right)\right]\ , \tag{10}$$

with $\beta_0=-4/3$ and the second contribution $C_{\mathsf{ii}}(y,Q^2)$ is given by

$$C_{\mathsf{ii}}^{(2,1)}(y,Q^2)=-\frac{\beta_0}{2}C^{(1,1)}(y,Q^2)\ . \tag{11}$$

In $C^{(2,1)}_{\mathsf{iii,iv}}(y,Q^2)$ there appear new subprocesses :

$$C^{(2,1)}_{\mathsf{iii}}(y,Q^2) = \int_{z_0^I}^1 dz P^0_{\gamma e}(z) J^I(z) \widetilde{C}^{(1,1)}_{e\gamma}(y,Q^2) \tag{12}$$

$$C^{(2,1)}_{\mathsf{iv}}(y,Q^2) = \int_{z_0^F}^1 dz P^0_{e\gamma}(z) J^F(z) \widetilde{C}^{(1,1)}_{\gamma e}(y,Q^2), \tag{13}$$

where $P^0_{\gamma e}$ and $P^0_{e\gamma}$ are LO off-diagonal splitting functions

$$P^0_{\gamma e} = \frac{1+(1-z)^2}{z}\ , \qquad P^0_{e\gamma} = z^2 + (1-z)^2\ . \tag{14}$$

$C^{(1,1)}_{e\gamma}(y,Q^2)$ and $C^{(1,1)}_{\gamma e}(y,Q^2)$ are defined in the same way as $C^{(1,1)}(y,Q^2)$ and their explicit expressions are given in [7].

The last contribution $C^{(2,1)}_{\mathsf{v}}(y,Q^2)$ is given by

$$\begin{aligned} C^{(2,1)}_{\mathsf{v}}(y,Q^2) &= \int_0^1 P^{1,NS,\mathrm{OM}}_{ee,S}(z) \Big[\theta\left(z - z_0^I\right) J^I(z) \widetilde{C}^{(0,0)}_I(y,Q^2) \\ &\quad -C^{(0,0)}(y,Q^2)\Big] + \int_{z_0^I}^1 P^{1,PS,\mathrm{OM}}_{ee,S}(z) J^I(z) \widetilde{C}^{(0,0)}_I(y,Q^2) \\ &+ \int_0^1 P^{1,NS,\mathrm{OM}}_{ee,T}(z) \Big[\theta\left(z - z_0^F\right) J^F(z) \widetilde{C}^{(0,0)}_F(y,Q^2) \\ &\quad -C^{(0,0)}(y,Q^2)\Big] + \int_{z_0^F}^1 P^{1,PS,\mathrm{OM}}_{ee,T}(z) J^F(z) \widetilde{C}^{(0,0)}_F(y,Q^2)\ . \end{aligned} \tag{15}$$

Here $P^1_{ee,S,T}(z)$ denote the space– and time–like NLO QED splitting functions of the non–singlet (NS) and pure–singlet (PS) channels in the on–mass–shell scheme which are obtained from the $\overline{\mathrm{MS}}$-scheme [8] by

$$P^{1,NS,\mathrm{OM}}_{ee,S,T}(z) = P^{1,NS,\overline{\mathrm{MS}}}_{ee,S,T}(z) + \frac{\beta_0}{2}\Gamma^{S,T,(1,1)}_{ee}(z)\ , \tag{16}$$

$$\Gamma^{S,T,(1,1)}_{ee}(z) = -2\left[\frac{1+z^2}{1-z}\left(\ln(1-z) + \frac{1}{2}\right)\right]\ , \tag{17}$$

and $P^{1,PS,\mathrm{OM}}_{ee,S,T}(z) = P^{1,PS,\overline{\mathrm{MS}}}_{ee,S,T}(z)$. We would like to remark that both the lepton–hadron interference term and the pure hadronic QED corrections, although apparently not widely known, are small. Already in $O(\alpha)$ their inclusion will only lead to a marginal change of the present result. In the case of the purely hadronic corrections details are explained e.g. in [1a].

4 Conclusions

We calculated the $O(\alpha^2 L)$ leptonic QED corrections to deep inelastic electron-nucleon scattering in the mixed variables. With the help of the RGE decomposition, the corrections are expressed as the convolutions of the splitting functions with the Born or 1–loop cross sections. This method generalizes earlier investigations for ISR in e^+e^- annihilation [6] and includes both space- and time-like splitting functions and Wilson coefficients.

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